Total sur 16

IE sur CA (30/03/2021) Tome

8

Exercice 1

$$A = \int_{-5}^{3} 2x^3 - 3x^2 + 7 \, dx$$

1 $$= \left[2\times\frac{1}{4}x^4 - 3\times\frac{1}{3}x^3 + 7x \right]_{-5}^{3} = \left[\frac{x^4}{2} - x^3 + 7x\right]_{-5}^{3}$$

1 $$= \frac{3^4}{2} - 3^3 + 7\times 3 - \left(\frac{(-5)^4}{2} - (-5)^3 + 7\times(-5)\right)$$

$$= 34{,}5 - 402{,}5$$

$$A = -368$$

1 $$B = \int_0^4 \frac{7x}{(x^2+3)^2} dx = \int_0^4 \frac{7}{2} \times \underbrace{\frac{2x}{(x^2+3)^2}}_{\text{forme } \frac{u'}{u^2} \text{ donc une primitive est } -\frac{1}{u}} dx$$

1 $$= \left[\frac{7}{2} \times \frac{-1}{x^2+3}\right]_0^4$$

0,5 $$= \frac{-7}{2(4^2+3)} - \frac{-7}{2(0^2+3)} = -\frac{7}{38} + \frac{7}{6}$$

$$B = \frac{56}{57}$$

$$C = \int_{-2}^{4} (-21t^2 + 28t)(-t^3 + 2t^2 + 1) \, dt$$

1 $$= \int_{-2}^{4} 7 \underbrace{(-3t^2+4t)(-t^3+2t^2+1)}_{\text{forme } u'u \text{ donc primitive : } \frac{1}{2}u^2} dt$$

1 $$= \left[\frac{7}{2}(-t^3+2t^2+1)^2\right]_{-2}^{4}$$

$$= \frac{7}{2}(-4^3 + 2\times 4^2 + 1)^2 - \frac{7}{2}(-(-2)^3 + 2\times(-2)^2 + 1)^2$$

1 $$= \frac{7}{2}\times 961 - \frac{7}{2}\times 289$$

$$C = 2352$$

7

Exercice 2

1 1) • $\dfrac{a}{(-3x+2)^2} + \dfrac{b}{-3x+2} = \dfrac{a+b(-3x+2)}{(-3x+2)^2} = \dfrac{a+2b-3bx}{(-3x+2)^2}$

• Donc : $f(x) = \dfrac{a}{(-3x+2)^2} + \dfrac{b}{(-3x+2)} \iff \begin{cases} a+2b = 1 \\ -3b = -12 \end{cases}$

1 $\iff \begin{cases} a = 1-2b \\ b = \dfrac{-12}{-3} = 4 \end{cases}$

$\iff \begin{cases} a = 1-2\times 4 = -7 \\ b = 4 \end{cases}$

D'où $a=-7$ et $b=4$.

2) $-3x+2 = 0 \iff x = -\dfrac{2}{-3} \iff x = \dfrac{2}{3}$

1

x	$-\infty$		$\frac{2}{3}$		$+\infty$
$-3x+2$		$+$	0	$-$	

d'où

x	-3		-1
$-3x+2$		$+$	

1 $I = \displaystyle\int_{-3}^{-1} \frac{-7}{(-3x+2)^2}\,dx + \int_{-3}^{-1} \frac{4}{-3x+2}\,dx$

2 $= \displaystyle\int_{-3}^{-1} \frac{7}{3} \times \underbrace{\frac{-3}{(-3x+2)^2}}_{\text{forme } \frac{u'}{u^2}}\,dx + \int_{-3}^{-1} \frac{-4}{3} \times \underbrace{\frac{-3}{-3x+2}}_{\text{forme } \frac{u'}{u} \text{ avec } u>0}\,dx$

$= \left[\dfrac{7}{3} \times \dfrac{-1}{-3x+2}\right]_{-3}^{-1} + \left[-\dfrac{4}{3}\ln(-3x+2)\right]_{-3}^{-1}$

1 $= \dfrac{7}{3} \times \dfrac{-1}{3+2} - \dfrac{7}{3} \times \dfrac{-1}{11} - \dfrac{4}{3}\ln 5 + \dfrac{4}{3}\ln 11$

$= \dfrac{-14}{55} + \dfrac{4}{3}\ln\left(\dfrac{11}{5}\right)$

1

Exercice 3

$\dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx.$